Collaborative Preferences in Multi-Objective Evolutionary Algorithms

Daniel Cinalli
UFF
drcinalli@ic.uff.br

Luis Martí
PUC-RIO
lmarti@ele.puc-rio.br

Nayat Sanchez-Pi
UERJ
nayat@iltc.br

Ana C. Bicharra Garcia
UFF
bicharra@ic.uff.br

ABSTRACT
This work presents a new approach of evolutionary multi-objective optimization algorithms augmented by collective intelligence interaction. In particular, we describe the extension of some well-known algorithms (NSGA-II, SMS-EMOA) to include collective online preferences and collaborative solutions into the optimization process. These innovative methods allow groups of decision makers to highlight the regions of Pareto frontier that are more relevant to them as to focus the search process mainly on those areas. Additionally, interactive and cooperative genetic algorithms work on users' collaborative preferences to improve the reference points and the population quality throughout the evolutionary progress. Rather than a unique or small group of decision makers provided with unilateral preferences, this paper promotes dynamic group preferences to aggregate consistent collective reference points and creative solutions to enhance multi-objective results. As part of this work we test the algorithms efficiency when face with some synthetic problem as well as a real-world case scenario.

Categories and Subject Descriptors
H.4 [**Information Systems Applications**]: Miscellaneous; J.4 [**Computer Applications**]: Social and behavioral sciences; G.1.6 [**Mathematics of Computing**]: Optimization

General Terms
Algorithms

Keywords
collective intelligence; preferences; reference points; evolutionary multi-objective optimization; collaboration

1. INTRODUCTION
Many real-life decision problems require managing trade-offs between multiple objectives. These problems can be posed as multi-objective optimization problems (MOPs) and they must consider more than one criteria to be simultaneously optimized [14]. In the general case, there is not a single optimal solution which optimizes all the objectives at the same time, but a set of points known as Pareto-optimal set that represents different trade-offs between the objectives. According to some *a priori* high-level preferences, a decision maker (DM) has to select which of those solutions are the ones to satisfy its needs.

MOPs tend to be NP-hard or NP-complete. Therefore, deterministic search techniques are usually unsuitable for this task. Metaheuristic and stochastic approaches are a viable alternative to handle this difficulty. Evolutionary algorithms (EAs) can find a finite population of solutions in one iteration run and disregard any particular shape of the underlying fitness landscape. It has prompted the creation of what has been called multi-objective optimization evolutionary algorithms (MOEAs) [7].

The approximation of the entire Pareto-optimal set in some application requires extensive time and computational resources. Preference information from the DM can be used for guiding the search to areas of interest and avoid regions which are of no relevance. MOEAs can use decision makers' contributions to improve the reference point and the population quality throughout the evolutionary process. Rather than a unique or small group of decision makers provided with unilateral preferences, a helpful input parameters can rely on the aggregation of vast and diverse masses of individual DM intelligence. The interaction of groups explores a diversity of answers and enhances multi-objective results.

The present work proposes and compares new preference-based interactive MOEAs augmented by collective intelligence (COIN) [22]. It introduces a new useful connection between these fields and extends some of the current state-of-the-art MOEAs. The hybrid algorithms aggregates consistent and online preferences from collective environment to the optimization process. Built upon the subjectivity of the crowds and human cognition, the intelligence of participatory actions addresses dynamic collaborative and creative intermediate solutions to overcome MOPs difficulties.

The combination of COIN in MOEAs aims at improving the quality of the obtained Pareto frontier approximation. Their results are driven not by one DM, but a group of people

SBSC10, November 4-6, 2015, Salvador, Bahia, Brazil.
ISSN: 2318-4132

that delimits their collective area of interest in the objective space. The new algorithms produce better solutions in the sense that they iteratively refine the search parameters and get users collaborations to generate more appropriated points in the final trade-off set.

The rest of this paper is organized as follows. Section 2 covers some required formal definitions of multi-objective optimization and collective intelligence field. The use and contributions of collective intelligence field in MOEAs are explored in Section 3. Section 4 presents the new algorithms based on interactive and collective intelligence techniques. Some results from benchmark problems and a resource placement case study are analyzed in Section 5. Finally, in Section 6, conclusive remarks and future work directions are put forward.

2. BASIC CONCEPTS

MOP can be stated as follows:

$$\left.\begin{array}{l} \text{minimize} \quad F(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), \ldots, f_k(\boldsymbol{x})\}, \\ \text{subject to } g_i(\boldsymbol{x}) \leq 0, \\ \qquad\qquad h_j(\boldsymbol{x}) = 0, \end{array}\right\} \tag{1}$$

where $\boldsymbol{x} = \langle x_1, \ldots, x_n \rangle \in \Omega$ is an n-dimensional decision variable. Thus, a MOP consists of k objectives, $m + p$ constraints, n decision variables and an evaluation function $\boldsymbol{F} : \Omega \to \mathcal{Z}$ that maps from the vector $\boldsymbol{x}$ to output vectors $\boldsymbol{a} = \langle a_1, \ldots, a_k \rangle$. The solution to this problem can be expressed by relying on the Pareto dominance relationship. A $\boldsymbol{x}$ is said to dominate $\boldsymbol{v}$ (denoted as $\boldsymbol{x} \prec \boldsymbol{v}$) iff $\forall i \in \{1, \ldots, n\}$, $x_i \leq v_i \wedge \exists i \in \{1, \ldots, n\}$ such that $x_i < v_i$.

A solution $\boldsymbol{x} \in \Omega$ is Pareto optimal if there does not exist another solution $\boldsymbol{x'} \in \Omega$ such that $F(\boldsymbol{x'}) \prec F(\boldsymbol{x})\}$. The Pareto-optimal set, P_S, is defined as $P_S = \{\boldsymbol{x} \in \Omega, \nexists \boldsymbol{x'} \in \Omega$ such that $F(\boldsymbol{x'}) \prec F(\boldsymbol{x})\}$. Similarly, the codomain of the set is known as the Pareto-optimal front, P_F [8].

Preferences are user-defined parameters and denote values or subjective impressions regarding the trade-offs points. It transforms qualitative feelings into quantitative values to bias the search during the optimization phase and restrict the objective space. In this sense, a reliable preference vector improves the trade-off answers obtained. Usually, a preference information c is represented by a set of criteria $c \in C$ and the boundaries are constrained by the *ideal* ($\boldsymbol{z}^*$), *utopian* ($\boldsymbol{z}^{**}$) and *nadir* ($\boldsymbol{z}^{nad}$) points.

The reference point approach [23] concentrates the search of non-dominated solutions near the selected point. It is based on the achievement scalarizing function that uses a reference point to capture the desired values of the objective functions. Let $\boldsymbol{z}^0$ be a reference point for an n-objective optimization problem of minimizing $F(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), ..., f_k(\boldsymbol{x})\}$, the reference point scalarizing function can be stated as follows:

$$\sigma\left(\boldsymbol{z}, \boldsymbol{z}^0, \boldsymbol{\lambda}, \rho\right) = \max_{i=1,\ldots,k} \left\{\lambda_i (z_i - z_i^0)\right\} + \rho \sum_{i=1}^{k} \lambda_i \left(z_i - z_i^0\right), \tag{2}$$

where $\boldsymbol{z} \in \mathcal{Z}$ is one objective vector, $\boldsymbol{z}^0 = \left\langle z_1^0, ..., z_k^0 \right\rangle$ is a reference point vector, σ is a mapping from $\mathbb{R}^k$ onto $\mathbb{R}$, $\boldsymbol{\lambda} = \langle \lambda_1, ..., \lambda_k \rangle$ is a scaling coefficients vector, and ρ is an arbitrary small positive number. Therefore, the achievement problem can be rebuilt as: min $\sigma\left(\boldsymbol{z}, \boldsymbol{z}^0, \boldsymbol{\lambda}, \rho\right)$.

2.1 Multi-Objective Evolutionary Optimization

MOEAs follow the common concepts of evolutionary algorithms (EA). In every generation t, they find a set of individuals non-dominated by the rest of the population. The parent and the offspring population sizes are μ and λ, respectively [16]. A space of individuals $i \in I$ represents the candidate solutions of a population P: $P(t) = (i_1(t), \ldots, i_\mu(t)) \in I^\mu$. A problem-specific fitness function $F : I \to \mathbb{R}$ measures if certain solution satisfies the objective functions. Some operators in charge of reproduction (crossover and mutation) and selection create offspring generations until a termination criterion is reached, such as: a candidate with acceptable quality; a previous computational constraint; neither non-dominated solutions comes out. After running a MOEA, the final population detains an approximation set (S) of all non-dominated solutions with finite size that is an appropriate representation of P_S.

Different performance indices are used to evaluate the quality of the Pareto front approximation. One of the techniques available is the hypervolume or S-metric indicator [15]. It is a quantitative metric that computes the region space covered by all non-dominated points. The hypervolume calculates the volume of the union of hypercubes a_i defined by a non-dominated point m_i and a reference point x_{ref} defined as:

$$\begin{aligned} S(M) &= \Lambda(\{\textstyle\bigcup_i a_i | m_i \in M\}) \\ &= \Lambda(\textstyle\bigcup_{m \in M} \{x | m \prec x \prec x_{ref}\}) \,. \end{aligned} \tag{3}$$

Pareto-optimal front coverage indicator, $D_{S \to P_F}$, is a proximity indicator [6] that defines the distance between an achieved approximation set S and their closest counterpart in the current Pareto-optimal front:

$$D_{S \to P_F}(S) = \frac{1}{|S|} \sum_{\boldsymbol{x} \in S} \min_{\boldsymbol{x'} \in P_S} \left\{d\left(\boldsymbol{x}, \boldsymbol{x'}\right)\right\}, \tag{4}$$

where d is the Euclidean distance between two points. If the Pareto-optimal front is continuous, a correct formulation of this indicator calls for a line integration over S. Small values of $D_{S \to P_F}$ indicate proximity to the Pareto-optimal front.

The variance (σ^2) is a statistical measure that expresses the dispersion of data. Instead of a good spread of solutions along P_F, the method proposed in this work wants to obtain subsets of solutions close to the collective reference point. In this context, a small variance means the individuals from the sample $Y = \{\boldsymbol{y}_1, \ldots, \boldsymbol{y}_N\}$ are clustered closely around the population mean (μ).

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (\boldsymbol{y_i} - \mu)^2 \tag{5}$$

In cases with more than one collective reference point ($\boldsymbol{z}^j$), points are clustered based on the closest distance to one of the reference points: $C_j = \{\boldsymbol{a} \in \mathbb{R}^k : \|\boldsymbol{a} - \boldsymbol{z}^j\| \leq \|\boldsymbol{a} - \boldsymbol{z}^i\|, \forall i\}$. Cluster C_j consists of all points for which $\boldsymbol{z}^j$ is the closest. The variance is calculated to each cluster separately.

2.2 Collective Intelligence

Since the beginning of 2000, the development of social network technologies and interactive online systems has promoted a broader understanding of the "intelligence" concept. A new phenomenon appeared based not only on the cognition of one individual, but also placed on a network of relationships with other people and the external world. The field known as collective intelligence (COIN) [20] is defined as the self-organized group intelligence arisen from participatory and collaboration actions of many individuals. Shared tasks or issues are handled by singular contributions in such a manner that their aggregation process creates better results and solves more problems than each particular contribution separately [22, 18]. This phenomenon develops a *sui generis* intelligence. It raises a global experience of collective attitudes without centralized control, bigger than its isolated pieces and sub-product of their combination.

COIN involves groups of individuals collaborating to create synergy and augment the human intellectual processes. A decision-making process over the Internet has to manage users' interactions. It must get valuable knowledge concealed or dispersed in the group, even when the participants are not specialized in the subject. This environment includes large and heterogeneous audiences that are mostly independent among each others. Therefore, the problem must be decomposed in tasks that sustain diversity and transient members' attendances to align the interest of crowds.

Handling collective intelligence means the combination of those tools and methods in a dynamic space of production to achieve an objective. As the purpose of this study is the enhancement of MOEAs through the use of the collective preferences, interactive genetic algorithms (IGA) are an appropriate technique to support this goal. IGA incorporates the evaluation of users on the candidates of evolutionary algorithms to solve problems whose optimization objectives are complex to be defined with exact functions [21]. Users' subjectivities are employed as fitness values to drive the search throughout the evolution process.

3. COIN IN MOEAS

The optimal frontier (P_F) might be extremely large or possibly infinite. However, DMs still must identify and retrieve the expected solutions to their demands from this trade-off set. Reference points and interactive techniques can be applied on MOEAs to support the DMs reaching a preferred sub-set of the front, instead of the entire front. Those methods use transitional results throughout the evolution process and improve the search with reference points or fitness function adjustments. But very few MOEA algorithms consider more than one user for reference point selection or evolutionary interaction. They neglect a collective scenario where many users could actively interact and take part of the decision process throughout the optimization.

COIN is a different level of abstraction and can be a special contribution to make MOEAs go beyond their reach. Human beings are used to multi-objective situations in their everyday lives. Complex scenarios that are hard for computer might be easier or natural for human's mind. People are able to improve the multi-objective algorithms with cognitive and subjective evaluation to find better solutions. Characteristics such as perception, strategy, weighting factors, agility, among others subjectivities might be introduced into the algorithm to generate a better pool of answers and enhance the optimization process.

This work proposes the integration of collective preferences to the optimization process of MOEAs. The main idea underlying this method is to drive the DM's search towards relevant regions in Pareto-optimal set and, also, promote the usage of COIN as a local search for new individuals. Following the Find-Fix-Verify method [3], some individuals from population are given to the users in order to get their update and feedback. This approach encourages the interaction of multiple participants and takes rational collaborations to improve the overall quality of EA population.

The algorithms indicated here (Section 4) have advantages over others preference-based MOEAs. They choose the reference points interactively. Their references are not defined *a-priori*, like the R-NSGA-II from Deb [12], nor indicated by the DM as the middle point in the Light Beam approach [10]. Rather, all the references are discovered online with the support of a genuine collective intelligence of many users.

A collective reference point produced by the interaction and aggregation of multiple opinions may provide a more accurate reference point than designed by only one DM (unilateral). A unique decision maker carries the risk of having mistaken guidelines or poor quality in terms of search parameter. Conversely, the synergy of actions and the heterogeneity inside collective environments develop creative resolutions based on the crowds' subjectivity and cognition.

There are plenty of examples where COIN techniques overcome individuals' results [20] and legitimize the attempt to incorporate this method within MOEAs. The Amazon Mechanical Turk site outsources digital tasks that are difficult for computers, but not for humans, such as: tagging images, writing product descriptions, identifying performers on music and so on. InnoCentive site hosts companies' problems and offers a cash prize to the one who presents the most preferred solution. Both initiatives harness collective ideas, elaborate global preferences to hit the target and outperform a design expert. Affinova delivers a service to companies who want to improve their innovation and marketing rates in consumer packaged goods, retail, financial services and design. Its platform empowers teams to develop ideas, collect consumer feedback and predict the best execution plan for them. Danone, a global food company, used their services to launch the Activia product line in USA and the result beat the initial forecast by four times [1]. Another example is the puzzle game about protein folding: Foldit; it uses the human brain's natural three-dimensional pattern matching to solve the problem of protein structure prediction. The highest scoring solutions are analysed by researchers and validated if applicable in real problems or not. Users in Foldit has already helped to decipher the crystal structure of the Mason-Pfizer monkey virus retroviral protease [19].

The free and easy-to-use application VizWiz [5] recruits web volunteers, including from Mechanical Turk marketplace, to help blind and visually impaired people. They send photos with recorded questions about text labels, colors or icons and get answers back in real time from online sources. Duolingo is a platform for practice and learning of several languages. Its gamified background motivates the users to earn experience points as they progress on dictations and lessons. The site uses crowdsourcing to discuss or fix grammar topics and translate real content from the web. MatLab, a famous matrix-based language for fast numeric computation, launched a coding contest which entries are scored and ranked online [17]. The MatLab challenges are manifold, such as: finding the n-th Fibonacci number as quickly as possible or planning routes for the rovers in Mars. All the entries are visible and the contestants can modify an existing one and submit it again as their own entry. This strategy promotes a kind of *co-opetition* (collaboration plus competition) that makes the solutions evolve by the collaboration of many people. Xprize, a non-profit organization, defines itself as an innovation engine and a catalyst for the benefit of humanity. This institution stimulates prize competitions on subjects like: global development and sustainable solutions; energy and climate change; life sciences and education. There is a monetary rewards for the winners, but the real intention is to encourage the global collectivity to invest the intellectual capital required for difficult problems.

4. ALGORITHMS

This section presents the new algorithms. They are extensions of the classical MOEAs: NSGA-II and SMS-EMOA. The main changes on the original methods are the incorporation of COIN into the selection procedure; the transformation of the continuous evolutionary process into an interactive one; and the adoption of reference points to drive the search towards relevant regions in Pareto-optimal front.

4.1 CI-NSGA-II

One of the new algorithms is a variation of NSGA-II [11]. The NSGA-II is a non-domination based genetic algorithm for multi-objective optimization. It adopts two main concepts: a density information for diversity and a fast non-dominated sorting in the population. The crowding distance uses the size of the largest cuboid enclosing two neighboring solutions to estimate the density of points in the front. Solutions with higher values of this measure are preferred rather than those in a more crowded region (smaller values) because they are better contributors to a uniformly spread-out Pareto front. The non-dominated sorting places each individual into a specific front such that the first front τ_1 is a non-dominant set, the second front τ_2 is dominated only by the individuals in τ_1 and so on. Each solution inside the front τ_n receives a rank equal to its non-domination level n.

The selection operator uses the rank (i_{rank}) and crowding distance (i_{dist}) in a binary tournament. The partial order $\prec_c$ between two individuals i and j, for example, prefers the minor domination rank if they are from different fronts or otherwise, the one with higher values of crowding distance. Then, crossover and mutation are applied to generate an offspring population.

$$i \prec_c j := i_{rank} < j_{rank} \vee (i_{rank} = j_{rank} \wedge i_{dist} > j_{dist}) \quad (6)$$

In algorithm 1, the new CI-NSGA-II converts the original NSGA-II into an interactive process. The subroutine *CollectiveContributions()* suspends the evolution progress and submits some individuals from population to the users' evaluation. The individuals received can be analyzed in two different ways by the user: a pairwise comparison allows the selection of the best candidate between two or more individuals; a dynamic game scenario stimulates the participant creativity to improve or produce new solutions. Both approaches discover online reference points, but only the last one generates alternatives for a rational improvement in the evolution process.

Algorithm 1 The Collective Intelligence NSGA-II.

$generation \leftarrow numgeneration$
$block \leftarrow subsetgeneration$ ▷ iteration interval
while $i < generation$ **do**
    while $block$ **do**
        $offspring \leftarrow$ **Tournament**(pop)
        $offspring \leftarrow$ **Crossover**$(offspring)$
        $offspring \leftarrow$ **Mutation**$(offspring)$
        $pop \leftarrow$ **COIN Selection**$(offspring)$
        $i++$
    end while
    $contributions \leftarrow$ **CollectiveContributions**$(front)$
    $pop \leftarrow contributions$
    $\Theta \leftarrow$ **ExpectationMaximization**$(contributions)$
    $pop \leftarrow$ **ReferencePoint Distance**(pop, Θ)
end while

After each collective interaction, the subroutine *ExpectationMaximization()* gets all the new individuals (or comparisons) and calculates the similarity of answers. The expectation maximization approach creates online reference points (Θ) for search optimization. A Gaussian Mixture model [2] is used to emulate the evaluation landscape of all participants' preferences. Figure 1 shows an example of three online reference points and the Gaussian distribution of their points from the well-known ZDT1 test suite [24].

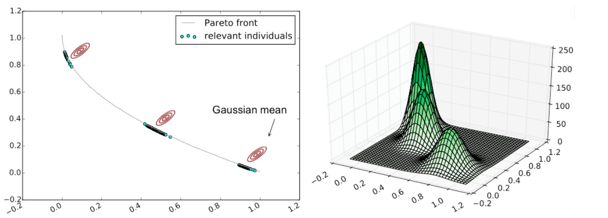


Figure 1: Online collective reference points.

The procedure *ReferencePointDistance()* calculates the minimum distance from each point in the population to the nearest collective reference points in Θ. This way, the point near the reference point is favoured and stored in the population.

The CI-NSGA-II develops a partial order similar to the NSGA-II procedure, but replaces the original crowding distance operator by the distance to collective reference points (i_{ref}). The *COIN Selection()* is based on this new partial order. Like NSGA-II, individuals with minor domination rank are preferred. But if they belong to the same front, the one with the closest reference point distance is used instead.

$$i \prec_c j := i_{rank} < j_{rank} \vee (i_{rank} = j_{rank} \wedge i_{ref} < j_{ref}) \quad (7)$$

4.2 CI-SMS-EMOA

The SMS-EMOA [4] is a steady-state algorithm that applies the non-dominated sorting as a ranking criterion and the hypervolume measure (S) as a selection operator.

After the non-domination ranking, the next step is to update the last front population, P_{worst}. the SMS-EMOA replaces the member with the minimum contribution to P_{worst} hypervolume by a new individual that increases the hypervolume covered by the population.

In algorithm 2, the new CI-SMS-EMOA converts the original SMS-EMOA into an interactive process. The subroutines *CollectiveContribution()* and *ExpectationMaximization()* work in the same way as the previous algorithm, CI-NSGA-II.

Algorithm 2 The Collective Intelligence SMS-EMOA.

$generation \leftarrow numgeneration$
$block \leftarrow subsetgeneration$ ▷ iteration interval
while $i < generation$ **do**
  while $block$ **do**
    $offspring \leftarrow$ **Tournament**(pop)
    $offspring \leftarrow$ **Crossover**$(offspring)$
    $offspring \leftarrow$ **Mutation**$(offspring)$
    $pop \leftarrow$ **COIN Selection**$(offspring)$
    $i++$
  end while
  $contributions \leftarrow$ **CollectiveContributions**$(front)$
  $pop \leftarrow contributions$
  $\Theta \leftarrow$ **ExpectationMaximization**$(contributions)$
  $pop \leftarrow$ **Hyper-RefPoint Distance**(pop, Θ, S)
end while

In the *COIN Selection()* operation, individuals with minor domination rank (i_{rank}) are preferred. If they belong to the same front, the one with the maximum contribution to the hypervolume of the set and the closest reference point distance (i_{ref}) is selected.

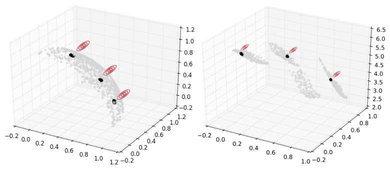
The procedure *Hype-RefPoint Distance()* gets the hypervolume contribution (S) and calculates the minimum distance from each solution in the population to the nearest collective reference points in Θ. This way, the solution with high hypervolume values and short reference point distance is favoured and stored in the new population.

5. EXPERIMENTAL RESULTS

This section presents some results of CI-NSGA-II and CI-SMS-EMOA. The multi-objective test problems ZDT [24] and DTLZ [13] have a known optimal front and can be used to benchmark the outcome of the algorithms. A real-world case is formally introduced afterwards and submitted to a COIN experiment.

5.1 Multi-Objective Test Problems

ZDTs and DTLZs are a set of well established scalable multi-objective test problems. Extensively used in MOEA studies, these benchmark problems were selected to analyse the behaviour of the proposed COIN MOEAs algorithms in the first moment. Each of these test functions knows *a priori* the exact shape and location of Pareto-optimal front. Their features cover different classes of MOPs: convex Pareto-optimal front, non-convex, non-contiguous convex parts and multimodal; and in the case of DTLZ are scalable to more than two objectives ($M > 2$). For those reasons, the test problems submit the new algorithms to distinct optimization difficulties and provide a broader comprehension of its working principles.



(a) DTLZ2 (b) DTLZ7

Figure 2: CI-NSGA-II results for DTLZ2 and DTLZ7 problems.

In the case of ZDTs and DTLZs test problems, the experiment emulates the collectivity by developing some virtual DMs (robots). The algorithms CI-NSGA-II and CI-SMS-EMOA suspends the evolution progress and submits some individuals from population to the robots' pairwise evaluation. Each robot has a predefined point in the objective space which will be used to direct robots' votes. The robot votes on a solution according to the closest distance between its predefined point and each of the two candidates. Another advantage is that one robot can perform multiple pairwise comparisons in one iteration.

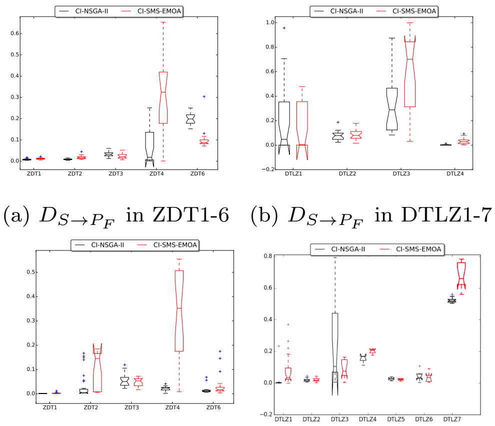
The robots create the collective reference points with a better reasoning than simply random choice. It is important to notice that the collective reference point is built on the similarity of answers after the Gaussian Mixture model and cannot be confused with the robots' predefined points.

Based on the best run of CI-NSGA-II in terms of Pareto-optimal front coverage indicator ($D_{S \to P_F}$) and variance (σ^2), figure 2 shows the relevant regions found in Pareto front to the DTLZ2 and DTLZ7 problems.

In addition to the Gaussian Mixture model, the K-means algorithm was implemented to bring a different clustering technique into the analysis. But the performance of Gaussian Mixture in these cases was consistently better than K-means. The front coverage ($D_{S \to P_F}$) and the variance (σ^2) indicators were used to measure the quality of the results. The hypervolume was not employed because their values depend on the spread of solutions in the whole Pareto front and the proposed algorithms, on contrary, aim to obtain subsets of solutions close to the collective reference points.

After 30 independent executions per EA on each test problem, figure 3 report the distribution of the front coverage and dispersion indicators in the form of box plots, respectively.

Although box plots allow a visual comparison of the results, it is necessary to go beyond reporting the descriptive statistics of the performance indicators. The need for comparing the performance of the algorithms when confronted with the different clustering techniques prompts the use of statistical



(a) $D_{S\to P_F}$ in ZDT1-6 (b) $D_{S\to P_F}$ in DTLZ1-7

(c) σ^2 in ZDT1-6 (d) σ^2 in DTLZ1-7

Figure 3: Distribution of $D_{S\to P_F}$ and σ^2 values for the tests.

Table 1: Results of the Conover-Inman statistical hypothesis tests. Green cells (+) denote cases where the algorithm in the row statistically was better than the one in the column. Cells marked in red (−) are cases where the method in the column yielded statistically better results when compared to the method in the row.

Problem		CI-NSGA-II	CI-SMS-EMOA
ZDT1	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
ZDT2	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
ZDT3	CI-NSGA-II	·	−
	CI-SMS-EMOA	+	·
ZDT4	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
ZDT6	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
DTLZ1	CI-NSGA-II	·	−
	CI-SMS-EMOA	+	·
DTLZ2	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
DTLZ3	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
DTLZ4	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
DTLZ5	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
DTLZ6	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·
DTLZ7	CI-NSGA-II	·	+
	CI-SMS-EMOA	−	·

tools in order to reach a valid judgement regarding the quality of the solutions and how different algorithms compare with each other.

The Conover-Inman procedure [9] is a non-parametric method particularly suited for this purpose. It can be applied in a pairwise manner to determine if the results of one algorithm were significantly better than those of the other. A significance level, α, of 0.05 was used for all tests. Table 1 contains the results of the statistical analysis, the CI-NSGA-II with Gaussian Mixture model consistently outperformed the CI-SMS-EMOA in these benchmarks. Concerning the convergence and dispersion measures, it was ranked best in all functions except for ZDT3 and DTLZ1. CI-NSGA-II and its collective reference points proved to be well matched for the range of ZDT and DTLZ test problems.

5.2 Resource Distribution problem

Many companies face problems of resource placement and assignment. A mining industry is one of the domain contexts where these problems are present. Those companies must extract valuable minerals or other geological materials from resource areas and allocate warehouses in such a way that optimizes its operational costs and production of collected resources. This general idea transforms the resource management into a multi-objective problem where one have to operate in an economic way and, at the same time, prioritize the performance or production.

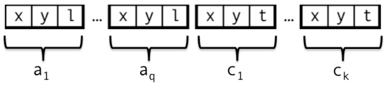


Figure 4: Chromosome encoding.

The problem —to put it in simple terms— has to find a good solution for positioning the processing units according the resource area. It is formally represented as:

$$\min \sum_{i=1}^{N}\sum_{j=1}^{M}\sigma_{ij}d_{ij} + \sum_{j=1}^{M}c_j\mu\,, \tag{8}$$

$$\max \sum_{i=1}^{N}\sum_{j=1}^{M}\sigma_{ij}v_j\,. \tag{9}$$

Let μ be the cost of one processing unit, v the productive capacity of one processing unit linked to one resource area, M a set of available positions to production units, N a set of available positions to resource area and D a distance matrix $(d_{ef})_{nxm}$, where $n \in N$ and $m \in M$. The decision variables are the processing unit c_j $(j \in M)$ that assumes 1 if it is placed at position j or 0 otherwise and σ_{ij} that assumes 1 if there is a link between the resource area at position $i \in N$ and the processing unit at position $j \in M$.

The processing unit is computationally represented as a tuple $c_i =< x, y, t >$; where $c_i \in C = \{c_1, ..., c_k\}$, t is the type of the unit, x and y are the Cartesian coordinates of the position. The resource area is represented by the tuple $a_i =< x, y, l >$; where $a_i \in A = \{a_1, ..., a_q\}$, l is a index that links the resource area a_i to the unit c_i. Thus, the chromosome encoding (figure 4) is the aggregation of these tuples regulated by q resource areas and k processing units.

Different constraints from real life and several new interdependencies among the variables might increase the search complexity of this MOP. Progressive articulation of preferences and collective intelligence can implement a dynamism not managed by *a priori* methods and enhance its efficiency. Therefore, the problem described is a candidate for this experiment due to some reasons: a) the objectives and decision variables are meaningful to the group, the problem is intuitive and allows an interaction with the crowd's cognition; b) incentive engines and gamification can be used to retain the users' interest on the interaction during the optimization; c) the problem can be decomposed in small blocks to be presented to the participants; d) the users' feed-backs can be parallelized in synchrony with the evolution of individuals in a MOEA.

In this context, the resource distribution problem was designed as a game where every player compete among themselves to obtain points and recognition of success. The game was implemented in a web-based platform and is open to all public. A gamification feature was implemented to motivate and promote the necessary alignment of goals to all the users in the collectivity. Gamification is the integration of game design elements and game engines in non-game con-

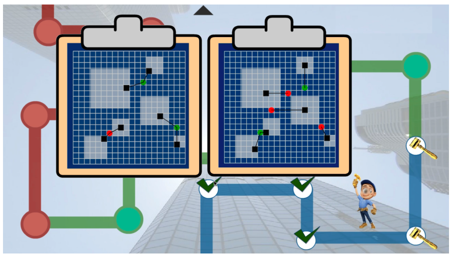


Figure 5: Gamification features and pairwise comparisons.

texts. This is usually intended to increase engagement of players, create gameful and playful user experiences, motivate them and set clear objectives to guide a cooperative or competitive behaviour.

The experiment was applied in two different computer labs: a brazilian professional education center with more than 30 students' attendance and a private company training room. After a certain number of iterations, the CI-NSGA-II algorithm interrupts the evolution process and asks the players for preferred individuals. There are two options in the game: pairwise comparison or free design mode.

In the pairwise comparison mode, the players must vote on the best candidate between two or more resource distribution scenarios (individuals from population). As votes on the scenarios happen, the Gaussian Mixture model calculates the collective reference point to restrict the search to relevant areas in Pareto front. The players who have chosen the individuals near the collective reference point receive a higher score. They compete at every evolution interval for choices around the collective mean. The game needs seven collective interactions to reach optimum points in the front. Figure 5 exhibits different phases of the game and the screen for pairwise comparisons of individuals.

In the free design mode, a dynamic game scenario encourages the creativity and cognition to produce new solutions. Some individuals from population are distributed to the players who have to fix and change their position arrangement. This game mode uses the collaboration to apply rational improvements in the quality of EA population.

Following the problem definition and constraints, the dynamic game scenario (figure 7) allows the creation of objects like trucks or warehouses, changing their arrangements and rebuilding their connections. For test purpose, there are two test environments: *open creation* and *only improvement*. The former opens the board for insert and deletion of objects, whereas the latter allows only the position and link rearrangement with the objects in the board.

The front coverage indicator, $D_{S \to P_F}$, measures the distance between the current approximation set S and the Pareto-optimal front. The proximity to the Pareto-optimal front $D_{S \to P_F} = 0.5$ is the criteria to stop the evolution and compare the algorithms. The values in table 2 and 3 represent the mean of all completed games.

The algorithm iteratively refines the search parameters and

Table 2: Performance for NSGA-II and CI-NSGA-II.

Environment	Performance Measure	NSGA-II	CI-NSGA-II	Generations
open creation	$D_{S \to P_F}$ σ^2	0,7 189	**0,5** **112**	120
only improvement	$D_{S \to P_F}$ σ^2	0,5 178	**0,5** **123**	145

Table 3: Performance for SMS-EMOA and CI-SMS-EMOA.

Environment	Performance Measure	SMS-EMOA	CI-SMS-EMOA	Generations
open creation	$D_{S \to P_F}$ σ^2	0,6 197	**0,5** **160**	143
only improvement	$D_{S \to P_F}$ σ^2	0,5 188	**0,5** **169**	151

adopts players collaborations to achieve more appropriated points in the final trade-off set. According to the tables 2 and 3, the CI-NSGA had a lower dispersion σ^2, which means the points are clustered closely around the collective reference point. It also required less generations to reach the convergence $D_{S \to P_F} = 0.5$ (figure 6). However, in the first generations of the evolution the distance between CI-NSGA-II front and the original NSGA-II front is greater than the last ones. It means that, for problems with low complexity, the original MOEAs reach the CI-NSGA-II progress. Although the CI-NSGA hit the proximity limit in the first place, the time spent on the human interaction was higher than the original MOEAs run.

Thus, in order to validate the efficiency of this novel approach, further studies on more complex scenarios intent to analyse the performance of CI-NSGA-II. In this context, the perspective is that CI-NSGA-II performs better, keeping the advantage of the original MOEAs and achieving the Pareto-optimal front.

This gamification aims to enhance the results of MOPs through COIN and support the DMs in the decision process. At the end of each game, only one scenario is presented to the players. Figure 7 shows the final solution from a single experiment game. After the collective reference points and users contributions in a game composed of six resource areas and two processing units, this candidate was progressively created with the support of users subjectivity and perception. From the group's point of view, it is the best alternative (winner candidate) and overcomes many others optimal points in the front.

6. FINAL REMARKS

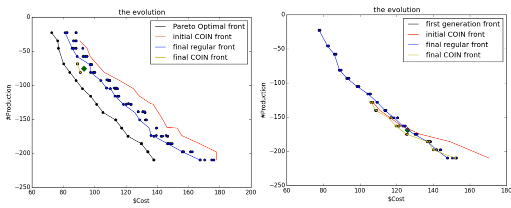


Figure 6: Convergence of CI-NSGA-II individuals.

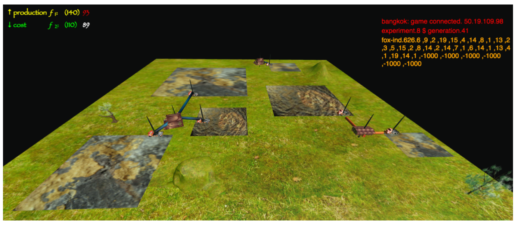


Figure 7: Evolutionary algorithm solution for six areas.

In this work we have introduced a novel interactive approach in multi-objective optimization evolutionary algorithms. The new algorithms CI-NSGA-II and CI-SMS-EMOA improve the successive stages of evolution via group's preferences and collaboration in a direct crowdsourcing fashion. The algorithms apprehend people's heterogeneity and common sense to guide the search through relevant regions of Pareto-optimal front and discover creative resolutions. The wisdom arisen from the diversity of many individuals is able to enhance MOEAs and overcome its difficulties.

A real-world case study regarding resource distribution was tested successfully against the algorithms. The multi-objective scenario was reproduced as a game and directed to a collective intelligence support. The ZDT and DTLZ benchmarking problems were also used to evaluate the new approaches. Their *a-priori* known Pareto-optimal front allowed the test of convergence and dispersion of points in the frontier. Results outlined the benefits of collective collaborations to unfold solutions designed by a group of people that is more intelligent when is working together.

In the near future, we plan to explore different features of the evolutionary process. We are particularly interested in more complex scenarios with many constraints and non-explicit objectives hidden in the problem. It is important to validate if the complexity of the environment will favour even more the integration of COIN in MOEAs. Furthermore, we intent to apply directional information from the collective reference points during the evolution process. This way, the technique can extract the intelligence of the crowds and, at the same time, minimize the interruptions of the algorithm.

References

[1] Success stories. `http://www.affinnova.com/clients/success-stories`, 2014. [Online; accessed 22-June-2014].

[2] D. Barber. *Bayesian reasoning and machine learning*. Cambridge University Press, 2012.

[3] M. S. Bernstein, G. Little, R. C. Miller, B. Hartmann, M. S. Ackerman, D. R. Karger, D. Crowell, and K. Panovich. Soylent: a word processor with a crowd inside. In *Proceedings of the 23nd annual ACM symposium on User interface software and technology*, pages 313–322. ACM, 2010.

[4] N. Beume, B. Naujoks, and M. Emmerich. Sms-emoa: Multiobjective selection based on dominated hypervolume. *European Journal of Operational Research*, 181(3):1653–1669, 2007.

[5] J. P. Bigham, C. Jayant, H. Ji, G. Little, A. Miller, R. C. Miller, R. Miller, A. Tatarowicz, B. White, S. White, et al. Vizwiz: nearly real-time answers to visual questions. In *Proceedings of the 23nd annual ACM symposium on User interface software and technology*, pages 333–342. ACM, 2010.

[6] P. A. Bosman and D. Thierens. The balance between proximity and diversity in multiobjective evolutionary algorithms. *Evolutionary Computation, IEEE Transactions on*, 7(2):174–188, 2003.

[7] C. C. Coello, G. B. Lamont, and D. A. Van Veldhuizen. *Evolutionary algorithms for solving multi-objective problems*. Springer, 2007.

[8] Y. Collette and P. Siarry. *Multiobjective optimization: Principles and case studies*. Springer, 2003.

[9] W. Conover. *Practical Nonparametric Statistics*. John Wiley and Sons, New York, 1999.

[10] K. Deb and A. Kumar. Light beam search based multi-objective optimization using evolutionary algorithms. In *Evolutionary Computation, 2007. CEC 2007. IEEE Congress on*, pages 2125–2132. IEEE, 2007.

[11] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: Nsga-ii. *Evolutionary Computation, IEEE Transactions on*, 6(2):182–197, 2002.

[12] K. Deb, J. Sundar, N. Udaya Bhaskara Rao, and S. Chaudhuri. Reference point based multi-objective optimization using evolutionary algorithms. *International Journal of Computational Intelligence Research*, 2(3):273–286, 2006.

[13] K. Deb, L. Thiele, M. Laumanns, and E. Zitzler. *Scalable test problems for evolutionary multiobjective optimization*. Springer, 2005.

[14] M. Ehrgott. *Multicriteria optimization*, volume 2. Springer, 2005.

[15] M. Emmerich, N. Beume, and B. Naujoks. An emo algorithm using the hypervolume measure as selection criterion. In *Evolutionary Multi-Criterion Optimization*, pages 62–76. Springer, 2005.

[16] M. Gendreau and J.-Y. Potvin. *Handbook of metaheuristics*, volume 2. Springer, 2010.

[17] N. Gulley. Patterns of innovation: a web-based matlab programming contest. In *CHI'01 Extended Abstracts on Human Factors in Computing Systems*, pages 337–338. ACM, 2001.

[18] F. Heylighen. Collective intelligence and its implementation on the web: algorithms to develop a collective mental map. *Computational & Mathematical Organization Theory*, 5(3):253–280, 1999.

[19] F. Khatib, F. DiMaio, S. Cooper, M. Kazmierczyk, M. Gilski, S. Krzywda, H. Zabranska, I. Pichova, J. Thompson, Z. Popović, et al. Crystal structure of a monomeric retroviral protease solved by protein folding game players. *Nature structural & molecular biology*, 18(10):1175–1177, 2011.

[20] T. W. Malone, R. Laubacher, and C. Dellarocas. Harnessing crowds: Mapping the genome of collective intelligence. 2009.

[21] X. Sun, L. Yang, D. Gong, and M. Li. Interactive genetic algorithm assisted with collective intelligence from group decision making. In *Evolutionary Computation (CEC), 2012 IEEE Congress on*, pages 1–8. IEEE, 2012.

[22] J. Surowiecki. *The wisdom of crowds*. Random House LLC, 2005.

[23] A. P. Wierzbicki. Reference point approaches and objective ranking. *Practical approaches to multi-objective optimization*, (06501), 2006.

[24] E. Zitzler, K. Deb, and L. Thiele. Comparison of multiobjective evolutionary algorithms: Empirical results. *Evolutionary computation*, 8(2):173–195, 2000.