Collective intelligence approaches in interactive evolutionary multi-objective optimization

DANIEL CINALLI*, *Universidade Federal Fluminense, 24210-346 Niterói, Brazil.*

LUIS MARTÍ**, *Universidade Federal Fluminense, 24210-346 Niterói, Brazil.*

NAYAT SANCHEZ-PI†, *Universidade do Estado do Rio de Janeiro, 20550-900 Rio de Janeiro, Brazil.*

ANA CRISTINA BICHARRA GARCIA††, *Universidade Federal do Estado de Rio de Janeiro, 22290-250 Rio de Janeiro, Brazil.*

Abstract

Evolutionary multi-objective optimization algorithms (EMOAs) have been successfully applied in many real-life problems. EMOAs approximate the set of trade-offs between multiple conflicting objectives, known as the Pareto optimal set. Reference point approaches can alleviate the optimization process by highlighting relevant areas of the Pareto set and support the decision makers to take the more confident evaluation. One important drawback of this approaches is that they require an in-depth knowledge of the problem being solved in order to function correctly. Collective intelligence has been put forward as an alternative to deal with situations like these. This paper extends some well-known EMOAs to incorporate collective preferences and interactive techniques. Similarly, two new preference-based multi-objective optimization performance indicators are introduced in order to analyze the results produced by the proposed algorithms in the comparative experiments carried out.

Keywords: collective intelligence, preferences, reference points, evolutionary multi-objective optimization algorithms

1 Introduction

Multi-objective optimization problems (MOPs) involve the simultaneous optimization a set of possibly contradictory objective functions. Formally, an MOP can be defined as the problem

$$min\mathbf{F}(\mathbf{x}) = \{f_1(\mathbf{x}), \ldots, f_k(\mathbf{x})\}, \text{ with } \mathbf{x} = \langle x_1, \ldots, x_n \rangle \in \Omega. \quad (1)$$

The solution of an MOP is the Pareto optimal set,

$$P_S := \{\mathbf{x} \in \Omega \mid \nexists \mathbf{y} \in \Omega : \mathbf{y} \prec \mathbf{x}\}, \quad (2)$$

*E-mail: drcinalli@gmail.com
**E-mail: lmarti@ic.uff.br
†E-mail: nayat@ime.uerj.br
††E-mail: bicharra@ic.uff.br

Vol. 28, No. 1, © The Author(s) 2020. Published by Oxford University Press. All rights reserved.
For permissions, please e-mail: journals.permission@oup.com.
Advance Access published 22 January 2020 doi:10.1093/jigpal/jzz074

that contains all the elements of Ω that not Pareto-dominated ($\prec$) by any other element [3]. Elements of P_S represent different trade-offs between the objective functions values. The projection of P_S through $\mathbf{F}()$ is known as the Pareto optimal front, P_F.

In the general case, MOPs (as all other optimization problems) are NP-hard [14]. That is why metaheuristic approaches have become the state-of-the-art methods to deal with them. The application of evolutionary computation to MOPs has prompted the creation of evolutionary multi-objective optimization algorithms (EMOAs) [3]. Most EMOAs approximate the P_S as a discrete set of points.

Having a Pareto optimal set that represents the solution of an MOP, a decision maker (DM) must identify which of those solutions are the ones that satisfy her/his preferences and should be realized in practice. This task can be rather complex and requires in-depth knowledge of the problem being solved, something that is often impossible in practical situations. Furthermore, it can be argued that educing and profiting from those preferences at early stages of the optimization process would allow the algorithm to focus only of areas of the Pareto optimal set that are of actual interest, thus reducing its computational footprint.

Because of these reasons, reference points [9] and interactive EMOAs [21] have be used to mitigate these inconveniences and support the DM in reaching a proper specification. Interactive techniques and reference points can be used to mitigate those inconveniences and steer the search for a suitable resolution in preferred areas of P_F. The combination of the evolutionary search process and DM preferences improves the population quality throughout the evolutionary process and leads to compromise solutions of practical interest. These approaches allow the optimization algorithm to reduce the search area and thus reaching satisfactory solutions at a lower computational cost.

In practice, however, optimization problems pose difficulties in defining *a priori* reference points or preferences. This mainly because a substantial problem domain knowledge is necessary on behalf of the DM. However, this contradicts the principles of evolutionary algorithms, as they are meant to be black-box optimizers that able to deal with highly uncertain problems and require as little as possible *a priori* knowledge. This situation is frequently aggravated as DMs with a sound knowledge of the application domain tend to be scarce or not cooperative.

In this regard, collective intelligence [15] reference points obtained by the interaction and aggregation of multiple opinions can be used to produce an accurate representation of preferences and, hence, reference points. This approach can also eliminate the unilateral choice bias that can be introduced by a single DM.

This work incorporates the online and interactive eduction of collective-based preferences into three existing EMOAs. In particular, we introduce a collective intelligence version of the nondomination sorting genetic algorithm (NSGA-II) [7], the improved strength Pareto evolutionary algorithm (SPEA2) [24] and the $\mathcal{S}$-metric selection evolutionary multiobjective algorithm (SMS-EMOA) [7].

Similarly, while working on this task it became evident the lack of adequate performance indicators that take into account preferences. Therefore, as part of this work, two new performance indicators are also introduced and used to evaluate the quality of the P_F approximation driven by the online collective preferences.

This paper is organized in the following manner. It proceeds with Section 2 that covers some required formal foundations regarding reference points and collective intelligence. Section 3 introduces the novel interactive EMOAs. Subsequently, Section 4 presents the new performance indicators that are to be used later on. After that, Section 5 analyzes the performance of the algorithms when face with benchmark problem. Finally, conclusive remarks and future work directions are put forward.

2 Foundations

Preferences are user-defined parameters and denote values or subjective impressions regarding the trade-offs points. It transforms qualitative feelings into quantitative values to bias the search during the optimization phase and restrict the objective space. In this sense, a reliable preference vector improves the trade-off answers obtained.

The reference point approach [23] concentrates the search of Pareto non-dominated solutions near a selected point. It is based on the achievement scalarizing function that uses a reference point to capture the desired values of the objective functions. Let $\mathbf{z}^0$ be a reference point for an n-objective optimization problem of minimizing $F(\mathbf{x}) = \{f_1(\mathbf{x}), ..., f_k(\mathbf{x})\}$, the reference point scalarizing function can be stated as follows:

$$\sigma\left(\mathbf{z}, \mathbf{z}^0, \boldsymbol{\lambda}, \rho\right) = \max_{i=1,...,k} \left\{\lambda_i\left(z_i - z_i^0\right)\right\} + \rho \sum_{i=1}^{k} \lambda_i\left(z_i - z_i^0\right), \tag{3}$$

where $\mathbf{z} \in \mathcal{Z}$ is one objective vector, $\mathbf{z}^0 = \left\langle z_1^0, ..., z_k^0 \right\rangle$ is a reference point vector, σ is a mapping from $\mathbf{R}^k$ onto $\mathbf{R}$, $\boldsymbol{\lambda} = \langle \lambda_1, ..., \lambda_k \rangle$ is a scaling coefficients vector and ρ is an arbitrary small positive number. Therefore, the achievement problem can be rebuilt as: min $\sigma\left(\mathbf{z}, \mathbf{z}^0, \boldsymbol{\lambda}, \rho\right)$.

A subset X of $\mathbf{R}^n$ is convex if for any two pair of solutions $x^1, x^2 \in X$ and $\alpha \in [0, 1]$, the following condition is true: $\alpha x^1 + (1 - \alpha)x^2 \in X$. The intersection of all the convex sets containing a given subset X of $\mathbf{R}^n$ is called the convex hull of X. The convex hull of a set of points is the smallest convex set that contains the points.

The *convex hull of individual minima* (CHIM) [5] is the set of points in $\mathbf{R}^k$ that are convex combinations of $F_i^* - F^*$. That is, letting $F_i^* = F(x_i^*), \forall i \in \{1, \ldots, k\}$, and $\boldsymbol{\Phi}$ is a pay-off matrix k xk whose the i^{th} column is $F_i^* - F^*$, the CHIM is defined as,

$$\mathcal{H} = \left\{ \boldsymbol{\Phi}\beta : \beta \in \mathbf{R}^k, \sum_{i=1}^{k} \beta_i = 1, \beta_i \geq 0 \right\}, \tag{4}$$

where x_i^* is the global minimizers of $f_i(\mathbf{x}), \forall i \in \{1, \ldots, k\}$.

2.1 Collective intelligence

Since the beginning of this century, the development of social network technologies and interactive online systems has promoted a broader understanding of the 'intelligence' concept. A new phenomenon appeared based not only on the cognition of one individual but also placed on a network of relationships with other people and the external world. The field is known as collective intelligence (COIN) [15] is defined as the self-organized group intelligence arisen from participatory and collaborative actions of many individuals. Shared tasks or issues are handled by singular contributions in such a manner that their aggregation process creates better results and solves more problems than each particular contribution separately.

Inside collective environment, contributions come from different people. Clustering algorithms distinguish the users with similar preferences to perform a cooperative evolution or decision making choice. A mixture model is a probabilistic model to reveal distributions of observations in the overall population. Given a data set $Y = \{\mathbf{y}_1, \ldots, \mathbf{y}_N\}$ where $\mathbf{y}_i$ is a d-dimensional vector measurement with the points created from density $p(\mathbf{y})$, a finite mixture model is defined as:

$$p\left(\mathbf{y}|\Theta\right) = \sum_{k=1}^{K} \alpha_k p_k\left(\mathbf{y}|z_k, \theta_k\right). \tag{5}$$

Let $K \geq 1$ be the number of components, $p_k(\mathbf{y}|z_k, \theta_k)$ be the mixture components where each k is a density or distribution over $p(\mathbf{y})$ and parameters θ_k, $\mathbf{z} = \langle z_1, \ldots, z_k \rangle$ be a K-ary random variable defining the identity of the mixture component that produced **y,** and $\alpha_k = p_k(z_k)$ are the mixture weights representing the probability that $\mathbf{y}$ was generated by component k. Hence, the parameters for a mixture model is $\Theta = \{\alpha_1, \ldots, \alpha_K, \theta_1, \ldots, \theta_K\}$, $1 \leq k \leq K$.

The central limit theorem [13] explains why many applications that are influenced by a large number of random factors have a probability density function that approximates a Gaussian distribution. Let Y be a sequence of random variables that are identically and independently distributed, with mean μ and variance σ^2. The distribution of the normalized sum $S_n = \frac{1}{\sqrt{n}}(\mathbf{y}_1 + \ldots + \mathbf{y}_N)$ approaches the Gaussian distribution, $G(\mu, \sigma^2)$, as $n \to \infty$.

In a Gaussian mixture model, each of the K components is a Gaussian density with parameters $\theta = \{\boldsymbol{\mu_k}, \boldsymbol{\Sigma_k}\}$, $\mathbf{y} \in \mathbb{R}^d$ and function as:

$$p_k(\mathbf{y}|\theta_k) = \frac{1}{(2\pi)^{d/2} |\boldsymbol{\Sigma_k}|^{1/2}} e^{-\frac{1}{2}(\mathbf{y}-\boldsymbol{\mu_k})^t \boldsymbol{\Sigma_k}^{-1}(\mathbf{y}-\boldsymbol{\mu_k})}. \tag{6}$$

The expectation maximization (EM) algorithm [11] for Gaussian mixture is a particular way of implementing the maximum likelihood estimation in probabilistic models with incomplete or missing data values. EM learns the parameters θ_k guessing a distribution for the unobserved data and finds the cluster to which a singular chromosome most likely belongs. It starts with an initial estimation of Θ and iterates between E-step and M-step of the algorithm to update Θ until convergence.

2.2 Interactice and prefecence-based EMOAs

Interactive genetic algorithms (IGAs) have successfully applied to get feedbacks of transitional results during the evolution process. IGA incorporates external evaluation to support problems whose optimization objectives are complex to be defined by *a priori* exact functions. It employs users' subjectivities as fitness values to drive the search engine.

Likewise, EMOAs can operate interactively and handle intermediate non-dominated solutions to the DM. Deb *et al*. [8] proposed an interactive EMOA approach termed PI-EMO. This technique changes progressively the value function after every few generations to lead the optimization on more preferred solutions. After t generations, the user analyses η (≥ 2) well-sparse non-dominated solutions and provides a complete or partial ranking of preference information about them. iEMOA/D [12], an interactive version of the decomposition-based multi-objective evolutionary algorithm, asks the DMs to analyse some current solutions and use their feedbacks to renew the preferred weight region in the following optimization. The algorithm IGAMII [1] applies fuzzy logic to simulate the human DM and relieve the constant interaction during the evolution. In BC-EMO [18], the support vector ranking algorithm is used to learn an approximation of the DM utility function.

3 Collective intelligence EMOAs

The purpose of this work is the enhancement of EMOAs through the use of the collective preferences; interactive genetic algorithms are an appropriate technique to support this goal. The new algorithms are extensions of the classical EMOAs: NSGA-II, SPEA2 and SMS-EMOA. The main changes on the original methods are the incorporation of COIN into the selection procedure, the transformation of the continuous evolutionary process into an interactive one and the adoption of reference points to drive the search towards relevant regions in Pareto optimal front.

3.1 CI-NSGA-II

One of the new algorithms is an improvement of NSGA-II [7]. The NSGA-II is a non-domination-based genetic algorithm for multi-objective optimization. It adopts two main concepts: a density information for diversity and a fast non-dominated sorting in the population. The crowding distance uses the size of the largest cuboid enclosing two neighboring solutions to estimate the density of points in the front. The non-dominated sorting places each individual into a specific front such that the first front τ_1 is a non-dominant set, the second front τ_2 is dominated only by the individuals in τ_1 and so on. Each solution inside the front τ_n receives a rank equal to its non-domination level n.

The selection operator uses the rank (i_{rank}) and crowding distance (i_{dist}) in a binary tournament. The partial order $\prec_c$ between two individuals i and j, e.g. prefers the minor domination rank if they are from different fronts or otherwise, the one with higher values of crowding distance.

$$i \prec_c j := i_{rank} < j_{rank} \vee (i_{rank} = j_{rank} \wedge i_{dist} > j_{dist}). \tag{7}$$

In Algorithm 1, the new CI-NSGA-II converts the original NSGA-II into an interactive process. The subroutine **CollectiveContributions()** suspends the evolution progress and submits some individuals from population to the users' evaluation. In this research, the individuals received can be analyzed in two different ways: (i) a pairwise comparison allows the selection of the best candidate between two or more individuals; (ii) a dynamic game scenario stimulates the participant creativity to improve or produce new individuals. Both approaches discover online reference points.

Algorithm 1 The Collective Intelligence NSGA-II.

```
generation ← numgeneration
block ← subsetgeneration                    ▷ iteration interval
while i < generation do
    while block do
        offspring ← Tournament(pop)
        offspring ← Crossover(offspring)
        offspring ← Mutation(offspring)
        pop ← COIN Selection(offspring)
        i++
    end while
    contributions ← CollectiveContributions(front)
    pop ← contributions
    Θ ← ExpectationMaximization(contributions)
    pop ← ReferencePoint Distance(pop, Θ)
end while
```

Inside collective environment, the contributions come from different people. Assuming the central limit theorem [13], the inputs have a distribution that is approximately Gaussian. Therefore, after each collective interaction, the subroutine **ExpectationMaximization()** gets the users' collaboration as a Gaussian mixture model to emulate the evaluation landscape of all participants' preferences.

The expectation maximization approach creates online reference points (Θ) for search optimization. Whether the user's collaboration is a simple vote on the best individual presented to him (pairwise comparison) or a complete re-edited individual, the clustering algorithm distinguishes the users with similar preferences to perform a cooperative evolution and a decision making choice through the collective reference points.

The procedure **ReferencePointDistance()** calculates the minimum distance from each point in the population to the nearest collective reference points in Θ. This way, the point near the reference point is favoured and stored in the new population. CI-NSGA-II develops a partial order similar to the NSGA-II procedure, but replaces the crowding distance operator by the distance to collective reference points (i_{ref}).

$$i \prec_c j := i_{rank} < j_{rank} \vee (i_{rank} = j_{rank} \wedge i_{ref} < j_{ref}). \tag{8}$$

This algorithm performs the **COIN Selection()** operation based on the new partial order. Like NSGA-II, individuals with minor domination rank are preferred. But if they belong to the same front, the one with the closest reference point distance is used instead.

CI-NSGA-II prioritizes the points close to the online collective reference point. The algorithm consumes preference information to explore satisfactory solutions for DMs.

3.2 CI-SMS-EMOA

The SMS-EMOA [2] is a steady-state algorithm that applies the non-dominated sorting as a ranking criterion and the hypervolume measure (S) as a selection operator.

After the non-domination ranking, the next step is to update the last front population, P_{worst}. It replaces the member with the minimum contribution to P_{worst} hypervolume by a new individual that increases the hypervolume covered by the population.

The new algorithm CI-SMS-EMOA converts the original SMS-EMOA into an interactive process. The **CollectiveContributions()** and **ExpectationMaximization()** subroutines have the same purpose and work as the CI-NSGA-II.

The selection operation, performed by the **COIN Selection()** procedure, prefers individuals with minor domination rank (i_{rank}). If they belong to the same front, the one with the maximum contribution to the hypervolume of the set and the closest reference point distance (i_{ref}) is selected.

A procedure **Hype-RefPoint Distance()** gets the hypervolume contribution (S) and calculates the minimum distance from each solution in the population to the nearest collective reference points in Θ. This way, the point with high hypervolume value and short reference point distance is favoured and stored in the new population.

3.3 CI-SPEA2

The improved Strength Pareto Evolutionary Algorithm (SPEA2) [24] developed a fitness assignment strategy based on the number of individuals that one solution dominates and it isdominated by. SPEA2 implements elitism by keeping an external population (archive) of size N. The archive preserves the best solutions since the beginning of the evolution.

The strength $ST(i)$ for each individual i is the number of population members it dominates: $ST(i) = |\{j : j \in P_t \oplus \bar{P}_t \wedge i \prec j\}|$, where $\oplus$ is the multiset union, P_t and $\bar{P}_t$ are the population and archive population at generation t, respectively. The fitness $F(i)$ for an individual i is given by the strength of its dominators: $F(i) = \sum ST(j)$, where $j \in P_t \vee \bar{P}_t, j \prec i$. High values of $F(i)$ means the individual i is dominated by many others and $F(i) = 0$ corresponds to a non-dominated individual.

SPEA2 uses a nearest density estimation technique, adapted from the kNN method, to distinguish individuals having the same fitness values. This density function is a function of the distance to the k-th nearest data point and it is added to the fitness function F.

In the new algorithm CI-SPEA2, subroutine **COIN Selection()** computes the strength of all individuals, and the non-dominated members are copied to the archive $\bar{P}_t$. The k-th nearest data

point used to calculate the original density function in SPEA2 was substituted by the collective reference points Θ. If the archive $\mid \bar{P}_t \mid \leq N$, the algorithm chooses the nearest individuals to the collective reference point until the archive size is reached. Otherwise, if $\mid \bar{P}_t \mid > N$, it removes the more distant ones proportionally to the number of individuals in each reference point cluster. This way, the archive keeps the same distribution of points around its reference points.

4 Performance indicators

Several performance indicators are used to evaluate the outcome sets of EMOAs. They measure the quality of the Pareto front approximation and allow comparison between different algorithms. There exist a variety of approaches that analyse the distribution of points in objective function space and the accuracy in terms of convergence.

The Pareto optimal front coverage indicator, $D_{S \to P_F}$, is a proximity indicator that defines the distance between an achieved approximation set S and their closest counterpart in the current Pareto optimal front:

$$D_{S \to P_F}(S) = \frac{1}{|S|} \sum_{\mathbf{x} \in S} \min_{\mathbf{x}' \in P_S} \left\{ d\left(\mathbf{x}, \mathbf{x}'\right) \right\}, \tag{9}$$

where d is the Euclidean distance between two points. Small values of $D_{S \to P_F}$ indicate proximity to the P_F.

However, some others indicators like coverage of two sets, diversity and the hypervolume could not be employed in this study. Because their values depend on the spread of solutions in the whole Pareto front and, on the contrary, the proposed algorithms aim to obtain subsets of solutions close to the collective reference points. There is a lack of performance indicators that focus only on the proportion of occupied area in P_F. For that reason, the following subsection presents two new ones.

4.1 Referential cluster variance indicator

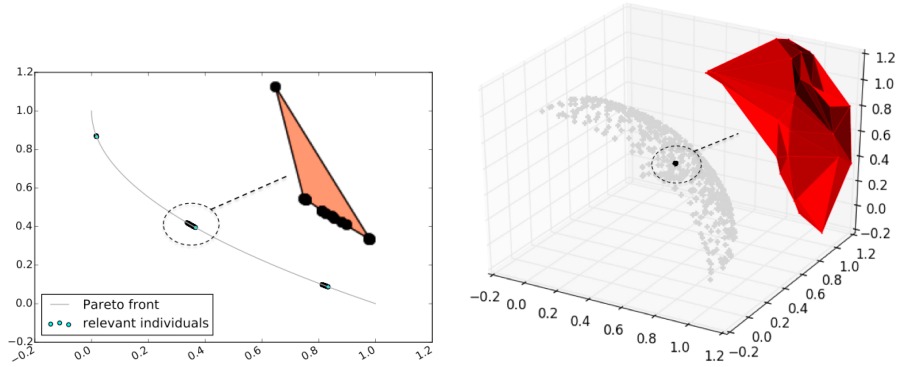
Instead of a good spread of solutions along P_F, the method proposed in this work wants to obtain subsets of solutions close to the collective reference point. In this context, a small cluster variance means the individuals from the sample $Y = \{\mathbf{y}_1, \ldots, \mathbf{y}_N\}$ of size N are clustered closely around the population mean (μ) or the reference point ($\mathbf{z}^0$). A low dispersion for a group of preferred points in P_F denotes a better efficiency of the approach tested. The referential cluster variance indicator κ is represented as follows:

$$\kappa = \frac{1}{N} \sum_{i=1}^{N} \left(\mathbf{y_i} - \mu\right)^2. \tag{10}$$

In cases with more than one collective reference point ($\mathbf{z}^j$), the points are clustered based on the closest distance to one of the reference points: $C_j = \{\mathbf{a} \in \mathbf{R}^k : \|\mathbf{a} - \mathbf{z}^j\| \leq \|\mathbf{a} - \mathbf{z}^i\|, \forall i\}$. Cluster C_j consists of all points for which $\mathbf{z}^j$ is the closest. The referential cluster variance is calculated to each cluster separately.

4.2 Convex hull volume indicator

Convex hull is a well-known geometric object widely used in various fields such as shape analysis, pattern recognition, geographical information systems, image processing, etc. It has been also applied in the multi-objective optimization scenario.



(a) ZDT1 (b) DTLZ3

FIGURE 1. Convex hull and alpha shape of non-dominated points after CI-NSGA-II iterations with one reference point.

The normal boundary intersection method (NBI) [5] projects elements of the CHIM towards the boundary $\partial\mathcal{Z}$ of the objective space $\mathcal{Z}$ through a normal vector N. The intersection point between $\partial\mathcal{Z}$ and N the normal pointing is a Pareto optimal point, if the P_F surface is convex. Martínez and Coello [16] introduced an archiving strategy based on the CHIM to find evenly distributed points along the P_F. Their convex hull multi-objective evolutionary algorithm (CH-EMOA) uses an archiving mechanism that stores non-dominated solutions that are orthogonal to each point of CHIM ($h \in \mathcal{H}$). Likewise, Shan-Fan *et al*. [20] presented an EMOA where the non-dominated solutions are picked out from dominated solutions by the quick convex hulls algorithm.

Wang *et al*. [22] proposed a convex hull-based multi-objective genetic programming (CH-MOGP) that follows similar strategies than SMS-EMOA and NSGA-II. But it uses convex hull-based sorting approach as an indicator schema to rank the individuals into different levels. Monfared [17] also employs convex hull concepts to elaborate a geometric ranking procedure for non-dominated comparisons in NSGA-II.

The idea of convex hull can be borrowed and applied as a performance indicator to measure the quality of the non-dominated points around the collective reference points. Connecting the closest final points to each reference point will produce a facet representation of the P_F. The volume of the convex hull can be used as a scalar indicator for the distribution of points in P_F. Small values of the hull volume (Ψ) indicate concentrate points.

Non-convex problems can use alpha shapes to determine a concave hull of theirs points in P_F. The alpha shape is a subgraph of the Delaunay triangulation. The value of alpha (α) controls the geometric design of the shape. For large α values the shape approaches to the boundary of the convex hull. On the other hand, as α decreases the shape shows more cavities.

Figure 1 illustrates the convex and non-convex enclosure for the non-dominated points generated by the CI-NSGA-II algorithm. The test problems used are the ZDT1 and DTLZ3, respectively (see Section 5).

5 Experimental results

This section presents some results of CI-NSGA-II, CI-SPEA2 and CI-SMS-EMOA. The scalable multi-objective test problems from the DTLZ and WFG problem sets [3] have a known optimal front

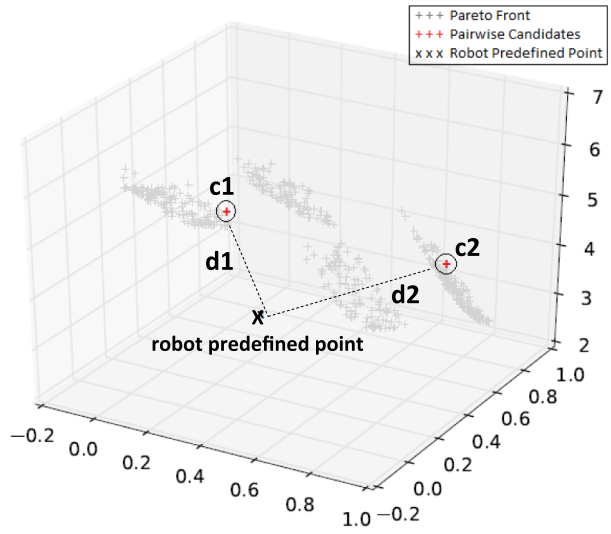


FIGURE 2. The robot's predefined point will choose candidate $c1$ because the distance $d1 < d2$.

and can be used to benchmark the outcome of the algorithms. Their features cover different classes of MOPs: convex P_F, non-contiguous convex parts, non-convex, multi-modal, etc. For those reasons, the test problems subject the new algorithms to distinct optimization difficulties and compare their results.

The experiment emulates the collectivity by developing some simulated DMs (robots). Each robot has a predefined point in the objective space that will be used to direct robots' votes in a pairwise comparison between two individuals from Pareto front. Therefore, the robot votes on a solution according to the closest distance between its predefined point and each of the two candidates.

Figure 2 illustrates candidates $c1$ and $c2$ with their respectively distances to the predefined point, $d1$ and $d2$. As $d1 < d2$, the robot would vote on $c1$. Robots create the collective reference points with a better reasoning strategy than simply random choice. It is important to notice that the collective reference point is built on the similarity of answers (votes) after the Gaussian mixture model and cannot be confused with the robots' predefined points.

Very few EMOAs consider more than one user for reference point selection or evolutionary interaction. They neglect a collective scenario where many users could actively interact and take part of the decision process throughout the optimization. Furthermore, the new algorithms choose the reference points interactively. The references are not defined *a priori*, like the R-NSGA-II [10], nor indicated by the DM as the middle point in the light beam approach [6]. Rather, all the references are discovered online with the support of a genuine collective intelligence of many users.

In this experiment, the robots abstract the collectivity within a controlled environment. So the algorithms can be tested, compared and better understood in their working principles. The quantity of online reference points is directly related to the number of k clusters in the Gaussian mixture model.

In cases where k is not previously defined, the experiment used the X-means algorithm [19] to learn k from the data. This algorithm searches different values of k and scores each clustering model using the Bayesian information criterion (BIC): $BIC(M_j) = \iota_j(D) - (p_j/2)\log R$, where D is the data set, M_j are models corresponding to solutions with different values of k, $\iota_j(D)$ is the log-likelihood

TABLE 1. Results of the Conover-Inman statistical hypothesis tests based on the mean values. Green cells (+) denote cases where the algorithm in the row statistically was better than the one in the column. Cells marked in red (−) are cases where the method in the column yielded statistically better results when compared to the method in the row.

	CI-NSGA-II	CI-SPEA2	CI-SMS-EMOA	R-NSGA-II	W-HYPE	CI-NSGA-II	CI-SPEA2	CI-SMS-EMOA	R-NSGA-II	W-HYPE	CI-NSGA-II	CI-SPEA2	CI-SMS-EMOA	R-NSGA-II	W-HYPE
	DTLZ2					**DTLZ3**					**DTLZ4**				
CI-NSGA-II	×	+	+	−	+	×	+	+	+	+	×	+	+	−	+
CI-SPEA2	−	×	+	−	+	−	×	+	+	+	−	×	+	−	+
CI-SMS-EMOA	−	−	×	−	+	−	−	×	−	+	−	−	×	−	+
R-NSGA-II	+	+	+	×	+	−	−	+	×	+	+	+	+	×	+
W-HYPE	−	−	−	−	×	−	−	−	−	×	−	−	−	−	×
	DTLZ5					**DTLZ6**					**DTLZ7**				
CI-NSGA-II	×	−	+	−	−	×	+	+	+	+	×	+	+	+	+
CI-SPEA2	+	×	+	−	−	−	×	−	−	+	−	×	−	−	+
CI-SMS-EMOA	−	−	×	−	−	−	+	×	−	+	−	+	×	+	+
R-NSGA-II	+	+	+	×	+	−	+	+	×	+	−	−	+	×	+
W-HYPE	+	+	+	−	×	−	−	−	−	×	−	−	−	−	×
	WFG1					**WFG2**					**WFG3**				
CI-NSGA-II	×	−	+	−	−	×	+	+	+	+	×	+	+	+	+
CI-SPEA2	+	×	+	+	−	−	×	+	+	−	−	×	−	+	+
CI-SMS-EMOA	−	−	×	−	−	−	−	×	+	−	−	+	×	+	+
R-NSGA-II	+	−	+	×	−	−	−	−	×	−	−	−	−	×	−
W-HYPE	+	+	+	+	×	−	+	+	+	×	−	−	−	+	
	WFG4					**WFG5**					**WFG6**				
CI-NSGA-II	×	+	+	+	+	×	+	+	−	+	×	−	−	−	+
CI-SPEA2	−	×	−	+	−	−	×	+	−	+	+	×	+	−	+
CI-SMS-EMOA	−	+	×	+	−	−	−	×	−	+	+	−	×	−	+
R-NSGA-II	−	−	−	×	−	+	+	+	×	+	+	+	+	×	+
W-HYPE	−	+	+	+	×	−	−	−	−	×	−	−	−	−	×
	WFG7					**WFG8**					**WFG9**				
CI-NSGA-II	×	−	+	+	−	×	+	−	+	+	×	+	+	+	+
CI-SPEA2	+	×	+	+	+	−	×	−	+	+	−	×	+	+	+
CI-SMS-EMOA	−	−	×	+	−	+	+	×	+	+	−	−	×	+	+
R-NSGA-II	−	−	−	×	−	−	−	−	×	−	−	−	−	×	+
W-HYPE	+	−	+	+	×	−	−	−	+	×	−	−	−	−	×

of the data set D according to model M_j, p_j is the number of parameters in M_j, and R is the number of points in the data set. X-means chooses the model with the best score.

The front coverage ($D_{S\to P_F}$), the referential cluster variance (κ) and the hull volume (Ψ) indicators were used to measure the quality of the algorithms. In addition to the Gaussian mixture model, the K-means algorithm was implemented to bring a different clustering technique into the analysis of the algorithms. But the performance of Gaussian mixture for these benchmarking cases was consistently better.

In order to provide a ground for comparison, two reference-point-based EMOAs are also included in the experiment: the R-NSGA-II and W-HYPE algorithms. Both algorithms use *a priori* the

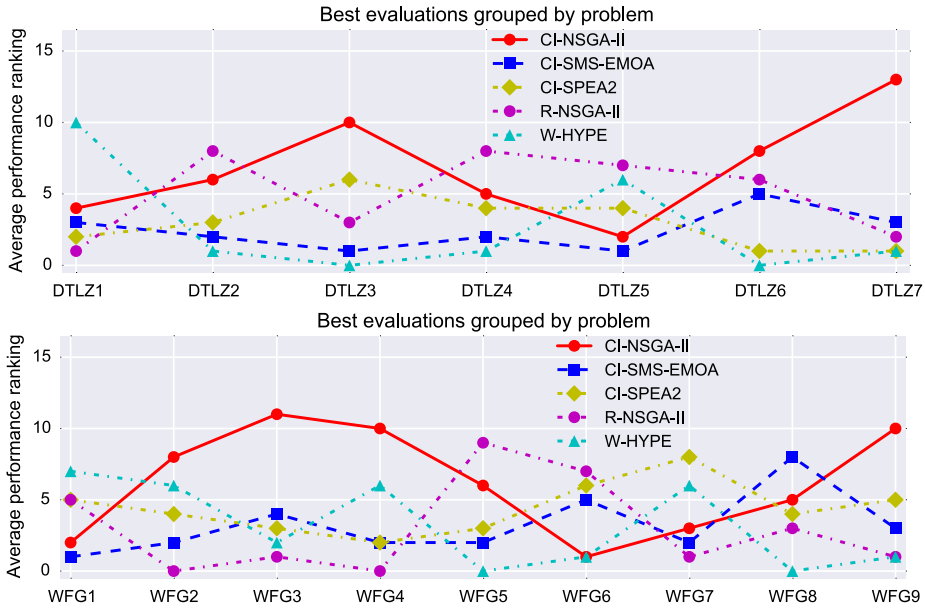


FIGURE 3. Average performance ranking across the DTLZ and WFG test problems.

reference points $\mathbf{z}^0 = (0.3, 0.3)$ and $\mathbf{z}^0 = (0.3, 0.3, 0.3)$ for the problems with two and three objectives, respectively.

For every algorithm/problem combination 30 experiment runs were performed. The resulting values pf the performance indicators were analyzed using the Conover-Inman procedure [4]. This is a non-parametric method for testing equality of population medians. It can be implemented in a pairwise manner to determine if the results of one algorithm were significantly better than those of the other. A significance level, α, of 0.05 was used for all tests. Table 1 contains the results of the statistical analysis for all the test problems based on the mean values.

The process of discovering the best algorithm is rather difficult as it implies cross-examining and comparing the results of their performance indicators.

Figure 3 present a more integrative representation by grouping their indicators. A higher value of average performance ranking implies that the algorithm consistently achieved lower values of the indicators being assessed.

In this case, lower values mean better convergence to P_F and higher concentration around the collective reference points. For a given set of algorithms $A_1, \ldots, A_K$, a set of P test problem instances $\Phi_1, \ldots, \Phi_P$, the function δ is defined as:

$$\delta\left(A_i, A_j, \Phi_p\right) = \begin{cases} 1 & \text{if } A_i \gg A_j \text{ solving } \Phi_p, \\ 0 & \text{otherwise} \end{cases} \tag{11}$$

where the relation $A_i \gg A_j$ defines if A_i is better than A_j when solving the problem instance Φ_p in terms of the performance indicators: $D_{S \to P_F}$, κ and Ψ. Relying on δ, the performance index $P_p(A_i)$ of a given algorithm A_i when solving Φ_p is then computed as: $P_p(A_i) = \sum_{j=1, j \neq i}^{K} \delta\left(A_i, A_j, \Phi_p\right)$.

The CI-NSGA-II with Gaussian mixture model consistently outperformed the others algorithms in most cases. Concerning the convergence indicator, CI-NSGA-II was ranked best in 16 of 21 test functions, except for DTLZ1, WFG6 and WFG8. The CI-SPEA2 and CI-SMS-EMOA have a similar performance on WFG tests. It is worth noticing that ZDT4 experiment demonstrated a premature convergence around the online reference points. A better control of the extent of obtained solutions must be investigated to avoid this behaviour.

In summary, the interactive EMOAs and their collective reference points proved to be well matched for the range of DTLZ and WFG test problems.

6 Final remarks

In this work, we have introduced new algorithms to improve the successive stages of evolution via dynamic group preferences. The interactive algorithms can apprehend people's heterogeneity and common sense to guide the search through relevant regions of Pareto optimal front.

The new approaches have been tested successfully in benchmark problems. Two different performance indicators were presented with the intention to measure the proportion of occupied area in P_F.

Currently, we have been exploring different features of the evolutionary process, such as: the usage of COIN as a local search for new individuals and opening the population for users' update to augment its quality.

Replacing the robots by human collaborators, this research has a simulated environment[1] that represents the facility location problem. In this context, individuals from the evolutionary algorithm population are distributed to the participants who have to fix and change the position arrangement or the resource distribution in the scenario. This simulated environment promotes the collaboration and supports rational improvements in the quality of the population. The algorithms —CI-NSGA-II, CI-SMS-EMOA and CI-SPEA2— introduced and compared here are used to iteratively refine the search parameters with collective preferences.

In the near future, we plan to work on creative solutions unfolded by the participants and possibly explore non-explicit objectives hidden in more complex scenario. Furthermore, we intent to apply directional information from the collective reference points during the evolution process. This way, the technique can extract the intelligence of the crowds and, at the same time, minimize the interruptions of the algorithm.

Acknowledgements

This work was supported in part by FAPERJ APQ1 Project 211.500/2015, FAPERJ APQ1 Project 211.451/2015, CNPq Universal 430082/2016-9, FAPERJ JCNE E-26/203.287/2017 and CNPq PQ 312792/2017-4.

References

[1] M. Babbar-Sebens and B. S. Minsker. Interactive genetic algorithm with mixed initiative interaction for multi-criteria ground water monitoring design. *Applied Soft Computing*, **12**, 182–195, 2012.

[1] http://playcanv.as/p/1ARj738G

[2] N. Beume, B. Naujoks and M. Emmerich. SMS-EMOA: multiobjective selection based on dominated hypervolume. *European Journal of Operational Research*, **181**, 1653–1669, 2007.
[3] C. A. Coello Coello, G. B. Lamont and D. A. Van Veldhuizen. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Springer, 2007.
[4] W. Conover. *Practical Nonparametric Statistics*. John Wiley and Sons, New York, 1999.
[5] I. Das and J. E. Dennis. Normal-boundary intersection: a new method for generating the pareto surface in nonlinear multicriteria optimization problems. *SIAM Journal on Optimization*, **8**, 631–657, 1998.
[6] K. Deb and A. Kumar. Light beam search based multi-objective optimization using evolutionary algorithms. In *IEEE Congress on Evolutionary Computation. CEC 2007*, pp. 2125–2132. IEEE, 2007.
[7] K. Deb, A. Pratap, S. Agarwal and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation*, **6**, 182–197, 2002.
[8] K. Deb, A. Sinha, P. J. Korhonen and J. Wallenius. An interactive evolutionary multiobjective optimization method based on progressively approximated value functions. *IEEE Transactions on Evolutionary Computation*, **14**, 723–739, 2010.
[9] K. Deb and J. Sundar. Reference point based multi-objective optimization using evolutionary algorithms. In *Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation - GECCO '06*, p. 635. ACM Press, New York, New York, USA, 2006.
[10] K. Deb, J. Sundar, N. Udaya Bhaskara Rao and S. Chaudhuri. Reference point based multi-objective optimization using evolutionary algorithms. *International Journal of Computational Intelligence Research*, **2**, 273–286, 2006.
[11] A. P. Dempster, N. M. Laird and D. B. Rubin. Maximum likelihood from incomplete data via the em algorithm. *Journal of the Royal Statistical Society. Series B (Methodological)*. **39**, 1–22, 1977.
[12] M. Gong, F. Liu, W. Zhang, L. Jiao and Q. Zhang. Interactive moea/d for multi-objective decision making. In *Proceedings of the 13th Anual Conference on Genetic and Evolutionary Computation*, pp. 721–728. ACM, 2011.
[13] C. M. Grinstead and J. L. Snell. Introduction to Probability. *American Mathematical Society*, 2012.
[14] R. Lassaigne and M. De Rougemont. *Logic and Complexity*. Springer Science & Business Media, 2012.
[15] T. W. Malone, R. Laubacher and C. N. Dellarocas. Harnessing Crowds: Mapping the Genome of Collective Intelligence (February 3, 2009). MIT Sloan Research Paper No. 4732-09. Available at SSRN: https://ssrn.com/abstract=1381502 or http://dx.doi.org/10.2139/ssrn.1381502, 2009.
[16] S. Z. Martínez and C. A. Coello Coello. An archiving strategy based on the convex hull of individual minima for MOEAs. In *2010 IEEE Congress on Evolutionary Computation (CEC)*, pp. 1–8. IEEE, 2010.
[17] M. D. Monfared, A. Mohades and J. Rezaei. Convex hull ranking algorithm for multi-objective evolutionary algorithms. *Scientia Iranica*, **18**, 1435–1442, 2011.
[18] D. Mukhlisullina, A. Passerini and R. Battiti. Learning to diversify in complex interactive multiobjective optimization. In *Proceedings of the 10th Metaheuristics International Conference (MIC 2013)*, 2013.
[19] D. Pelleg and A. W. Moore. X-means: extending k-means with efficient estimation of the number of clusters. In *Proceedings of the Seventeenth International Conference on Machine Learning, ICML'00*, pp. 727–734. Morgan Kaufmann Publishers Inc, San Francisco, CA, USA, 2000.

[20] J. Shan-Fan, S.-W. Xiong and J. Zhuo-Wang. The multi-objective differential evolution algorithm based on quick convex hull algorithms. In *Fifth International Conference on Natural Computation, 2009. ICNC'09*, ed, vol. 4, pp. 469–473. IEEE, 2009.
[21] H. Takagi. Interactive evolutionary computation: fusion of the capabilities of EC optimization and human evaluation. *Proceedings of the IEEE*, **89**, 1275–1296, 2001.
[22] P. Wang, M. Emmerich, R. Li, K. Tang, T. Back and X. Yao. Convex hull-based multiobjective genetic programming for maximizing receiver operating characteristic performance. In *IEEE Transactions on Evolutionary Computation*, vol. 19, pp. 188–200, 4, 2015.
[23] A. P. Wierzbicki. Reference point approaches and objective ranking. In *Practical Approaches to Multi-Objective Optimization, number 06501 in Dagstuhl Seminar Proceedings, Dagstuhl*, J. Branke, K. Deb, K. Miettinen and R. Slowinski, eds. Internationales Begegnungs- und Forschungszentrum für Informatik (IBFI), Schloss Dagstuhl, Germany, Germany, 2007.
[24] E. Zitzler, M. Laumanns and L. Thiele. SPEA2: improving the strength Pareto evolutionary algorithm. *TIK-Report*, **103**, 2001.

Received 20 October 2017